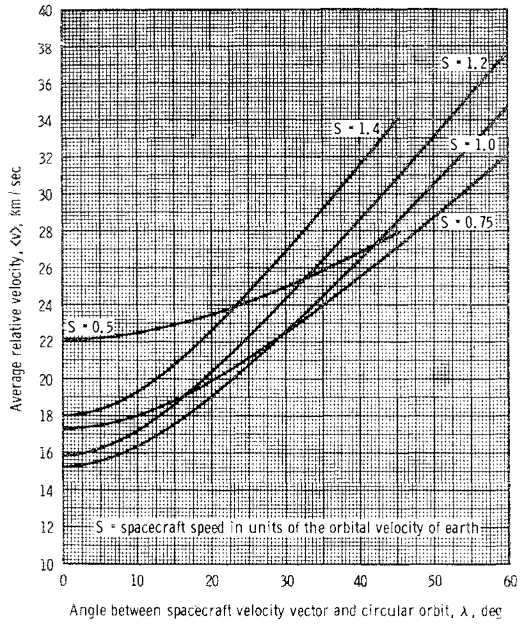


**Fig. 3 Average meteoroid velocity relative to a spacecraft at 1.0 a.u.**

in order to determine the velocity distribution relative to a massless earth; $v_G$ is the meteoroid velocity relative to a massless earth. The resulting geocentric velocity distribution is shown in Fig. 2, with an average velocity of 15.2 km/sec.

An incidental result of producing Figs. 1 and 2 is that the ratio of the sum of all weighting factors used to produce Fig. 1 to the sum of the weighting factors used in Fig. 2 is equal to the ratio of the meteor flux on the surface of the earth to the meteoroid flux outside the gravitational sphere of influence of the earth. This ratio is equal to 2.1.

### Average Velocity Relative to a Noncircular Orbit

Assume that a spacecraft is at 1 a.u. from the sun and outside the gravitational sphere of influence of the earth and that the spacecraft has a heliocentric speed of $S$ (in units of the orbital velocity of the earth). The velocity vector of the spacecraft is in the ecliptic plane, making an angle $\lambda$ with the orbital path of the earth. Then from the semimajor axis $a$, inclination $i$, and eccentricity $e$ of each meteor as given in Ref. 1, the intersection velocity $v$ can be computed by using standard orbital mechanics. However, the weighting factor $v/v_G$ must be added to consider the differences in the probability of collision for circular and noncircular orbits.[7] Thus, the total weighting factor becomes $vv_G/v_\infty^{5.5}$.

Velocity distributions were obtained for various values of $S$ and $\lambda$, and the average velocity $\langle v \rangle$ was then obtained for each distribution. Figure 3 gives $\langle v \rangle$ as a function of $\lambda$ for various values of $S$. At $S = 1.0$ and $\lambda = 0.0$, $\langle v \rangle$ is the same on Fig. 3 as on Fig. 2.

Figure 3 can be used to determine $\langle v \rangle$ for any distance from the sun by making the assumption that the distribution of $a/R$, $e$, and $i$ for meteoroids is independent of the distance from the sun $R$. In this general case, $S$ is redefined to be the ratio of the spacecraft speed at $R$ to the circular orbit speed at $R$, and $\lambda$ is the angle between the spacecraft velocity vector and a circular orbit at $R$. The average relative meteoroid velocity will then vary as $R^{-0.5}$ for constant values of $S$ and $\lambda$. For example, if a spacecraft is in orbit around the sun at $R = 1.25$ a.u. and if the orbit has a perihelion of 1.0 a.u. and an aphelion of 1.5 a.u., then $S = 1.0$, $\lambda = 11.5°$, and (from Fig. 3) $\langle v \rangle = 16.6/(1.25)^{1/2} = 14.8$ km/sec.

It is unlikely that a spacecraft would have values of $S$ greater than 1.4 or less than 0.5. A value of $S = (2)^{1/2}$ corresponds to a parabolic orbit, and $S = 0.5$ is the speed of a spacecraft at the aphelion of a highly elliptical transfer orbit having an eccentricity of 0.75. A value of $S = 1.0$ corresponds to the spacecraft speed when the spacecraft is at a distance from the sun equal to the semimajor axis of the transfer orbit. Thus, Fig. 3 can be used to determine the average relative meteoroid velocity for most interplanetary spacecraft.

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# Interpretation of Scalar Correlation Coefficients

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RECENTLY published measurements of the properties of the charged particle fluctuations in a turbulent jet,[1] have shown that the transverse and longitudinal correlation coefficients of the electron density fluctuations are related through an equation specifying the isotropy of the velocity fluctuations. In this Note, some implications of these observations will be examined. In particular, it will be shown that the implied character of the velocity fluctuations give a consistent description of the measured optimum space-time correlation and the measured spectrum of a scattered microwave signal.

The correlation coefficient of electron-density fluctuations, for points separated by $\boldsymbol{\delta}$, is defined as

$$R(\boldsymbol{\delta}) = \frac{\langle \tilde{n}(\mathbf{r})\tilde{n}(\mathbf{r}+\boldsymbol{\delta})\rangle}{n'(\mathbf{r})n'(\mathbf{r}+\boldsymbol{\delta})} \tag{1}$$

where $\tilde{n}$ is the fluctuating component of the electron density; the angular bracket denotes a time or volume average; and the prime designates the root mean square value of the fluctuating electron density. The measurements of the charged particle correlation coefficients were made in the exhaust of a turbulent flame. An extended turbulent plasma was formed by seeding a premixed ethylene-oxygen flame in a combustion chamber and then exhausting it through a 2.5-cm-

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diam expansion nozzle into a low-pressure vessel with dimensions of 1.2 × 1.2 × 2.4 m. The principal combustion products are carbon monoxide, carbon dioxide, and water vapor. The electron density is controlled by varying the amount of seed material (potassium chloride) inserted into the combustion chamber. The flow conditions for the current experiments are somewhat modified from those previously described.[1] The ambient pressure for the current experiments is 6 torr rather than 15 torr, and the mass flow is 3 g/sec rather than 1 g/sec. These changes resulted in higher electron densities for an electromagnetic scatter experiment. For the current experimental conditions, the plasma is not in chemical equilibrium, whereas for the previous conditions the plasma was very nearly in chemical equilibrium at the highest electron density levels. (The term chemical equilibrium is used to indicate that the Saha equation is valid.) The effect of the departure from chemical equilibrium is to reduce the axial inhomogeneity in electron density.

The measurement of the charged particle correlations were accomplished using an electrostatic probe biased for ion collection and a signal correlator.[1] The mean probe-current dependence on mean electron density was calibrated against a microwave interferometer.[2] The linear dependence of probe-current fluctuations was determined from a calibration against a microwave impedance probe having equally high spatial resolution.[1] The ion-density statistics are identical to the electron-density statistics because the Debye length is always small compared to all lengths of interest.

Representative measurements of the longitudinal (with respect to the jet axis) and transverse correlation coefficients are shown in Fig. 1 and Fig. 2, respectively. The solid line in Fig. 1 is the function

$$R_L(\delta) = [3 \exp - (\delta/1.5)] - 2[\exp - (\delta/1.0)] \quad (2)$$

(where $\delta$ is in cm) which is seen to be a good approximation to the measured correlation data points. The transverse correlation coefficient is observed, in Fig. 2, to be a good fit to the function

$$R_T(\delta) = 3\left[\left(1 - \frac{\delta}{2(1.5)}\right) \exp - \left(\frac{\delta}{1.5}\right)\right] - 2\left[\left(1 - \frac{\delta}{2(1.0)}\right) \exp - \left(\frac{\delta}{1.0}\right)\right] \quad (3)$$

so that Eqs. (2) and (3) satisfy the relation

$$R_T = R_L + (\delta/2)dR_L/d\delta \quad (4)$$

The relation between the longitudinal and transverse correlation coefficients, given in Eq. (4), was satisfied by the experimental data in all regions where the turbulent flow was fully developed. It should be noted that Eq. (4) was also satisfied in the previous set of experimental conditions for which the plasma was very nearly in chemical equilibrium when the electron density was high.

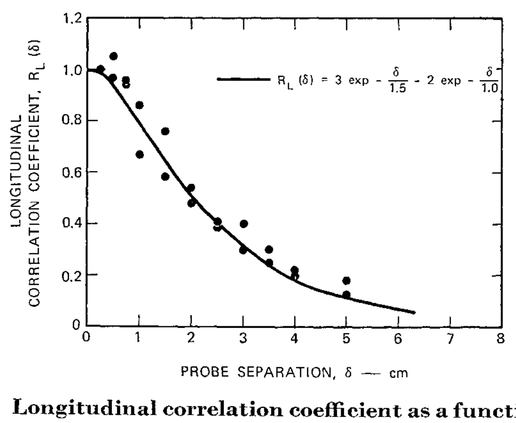


**Fig. 1 Longitudinal correlation coefficient as a function of probe separation.**

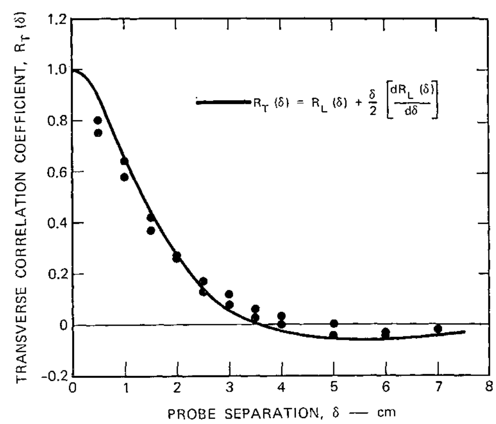


**Fig. 2 Transverse correlation coefficient as a function of probe separation.**

Batchelor[3] has demonstrated that the condition for isotropy of the turbulent velocity fluctuations is that the correlation of any component of velocity should satisfy Eq. (4). In that case, longitudinal and transverse separation of test points is referred to the particular component of velocity being measured. When measurements are made on the random velocity component parallel to the mean velocity, the subscripts $L$ and $T$ have the same meaning as when used here.

When the velocity fluctuations are isotropic, the parallel velocity components will satisfy Eq. (4). Since velocity fluctuations are so important, the turbulent flow is then usually designated as isotropic flow without regard to the correlation of scalar quantities. However, if scalar quantities serve as markers for the velocity fluctuations, they will also satisfy Eq. (4) and then the distribution of these scalar quantities will be anisotropic. [Isotropy of scalar quantities requires that $R_T(\delta) = R_L(\delta)$.] As a consequence, the turbulent field is, at once, isotropic in the velocity fluctuations and anisotropic in the scalar quantity fluctuations.

The most simple way, but not necessarily the only way, for scalar quantities such as charged particle density ($n_+$), neutral particle density ($p$), or gas temperature ($T_e$), to have the same correlation coefficient as the fluctuating component of velocity parallel to the mean flow ($\mathbf{u}$), is for them to be strongly linearly related. In particular, under certain flow conditions, this linear relationship would be expected, and this makes a linear relationship plausible for the plasma jet although it is not possible to establish the linearity with complete certainty.

The linear relationship can be established by linearly relating the charged particle fluctuations to temperature or neutral particle density fluctuations and then linearly relating these fluctuations to the velocity fluctuations. The neutral particle density fluctuations or the temperature fluctuations can be linearly related to the fluctuation of the longitudinal velocity component when the turbulent flow has small turbulent intensity and when the fluctuations in total enthalpy are negligible (the strong Reynolds analogy).[4] The approximate similarity of scalar and longitudinal velocity fluctuations has been observed by Kistler[5] in a boundary layer and Demetriades in a wake.[6] Corrsin and Uberoi have measured temperature and velocity fluctuations in a jet.[7] They have measured the spectra of these two quantities and found them to be approximately the same except at small wave numbers (which means the correlation lengths for temperature and velocity fluctuations will be different).

Several situations could establish a strong linear relation between the charged particle density and temperature. When the chemistry of the plasma flow is frozen, the charged particle density fluctuations would be directly proportional to the neutral particle density fluctuations (and hence, inversely proportional to the temperature fluctuations). Secondly, when the flow is in chemical equilibrium, the Saha

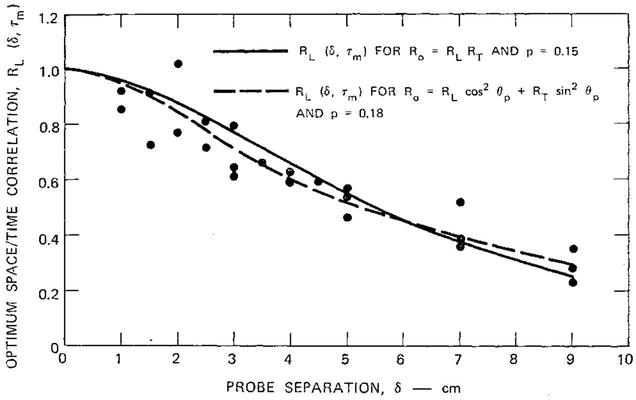


**Fig. 3 Optimized correlation coefficient as a function of probe separation.**

equation indicates that the charged particle density fluctuations vary directly with temperature fluctuations when the intensity of temperature fluctuations is small. An even stronger relationship shows that $n_+$ and $T_e$ will have linearly related correlation functions.[8] In this case, the correlation functions will be linearly related when the condition on the strength of the temperature fluctuations is given by

$$[T'_e/\langle T_e\rangle] < [\langle T_e\rangle/5800\ V_i] \tag{5}$$

where $V_i$ is the ionization potential in volts of the ionizing species. For potassium and $\langle T_e\rangle = 2{,}500°\mathrm{K}$, this condition requires $(T'_e/\langle T_e\rangle)$ to be less than 0.1.

That the flow of this experiment satisfies enough of the foregoing requirements to linearly relate charged particle density fluctuations and velocity fluctuations is problematical. It should be noted, however, that the longitudinal and transverse electron density correlation coefficients were experimentally observed to be related by Eq. (4), independent of whether the chemistry was out of equilibrium (as for the current experiment) or in equilibrium (as for the previous experimental conditions). In what follows, it shall be assumed that the correlations for charged particle fluctuations and velocity fluctuations (for **u**) are identical. Based on this assumption, calculations of the optimum correlation coefficient shall be made and compared with further experimental results.

An analysis based upon the theoretical models of Favre[9] and Lane[10] relates measured spatial correlations to the optimum correlation coefficient, $R_{\mathrm{opt}}(\delta, \tau)$, which is the maximum correlation observed for longitudinal probe separation $\delta$ when the time delay $\tau$, in the upstream channel, has been adjusted for maximum correlation. Lane has expressed the optimum longitudinal time/space correlation coefficient, in terms of the measured spatial correlation as

$$R_{\mathrm{opt}}(\delta,\tau) = \int\int_{-\infty}^{\infty}\int R(\xi)|2\pi p^2\delta^2|^{-3/2} \exp - \left[\frac{\xi^2}{2p^2\delta^2}\right] d\xi \tag{6}$$

where the turbulent velocity components have been assumed to be equal in magnitude (i.e., the velocity fluctuations are isotropic) and $p$ is defined to be $(u'/\bar{U})$. The correlation for arbitrary separation given by the vector $\xi$, $R(\xi)$, can be written in terms of the measured longitudinal and transverse correlation coefficients in two different forms;

$$R(\xi) = R_L(\xi_x)R_T(\xi_r) \tag{7}$$

or

$$R(\xi) = R_L(r)\cos^2\theta_p + R_T(r)\sin^2\theta_p \tag{8}$$

where $\xi_x$ and $\xi_r$ [Eq. (7)] are the longitudinal and radial variables in cylindrical coordinates, $r$ [Eq. (8)] is the radial variable in spherical coordinates, and $\theta_p$ is the polar angle of $\xi$ with respect to the jet axis. The latter expression for $R(\xi)$ [Eq. (8)] is that used by Lane and Favre. Equation (6) is integrable when $R_L$ is an exponential, or the sum of exponential terms, of the form $R_L = \exp - (\delta/\Lambda)$. Thus, for the alternate forms of $R(\xi)$, and when $R_L$ and $R_T$ are related through Eq. (4). The optimum correlation becomes

$$R_{\mathrm{opt}}(\delta,\tau) = (\exp b^2)(\mathrm{erfc}\, b) \times \{(1 + b^2) - b(\pi)^{1/2}[(\tfrac{3}{2}) + b^2](\mathrm{erfc}\, b)(\exp b^2)\} \tag{9}$$

or

$$R_{\mathrm{opt}}(\delta,\tau) = [1 + 4b^2 + (\tfrac{4}{3})b^4] \exp b^2 \,\mathrm{erfc}\, b - [b/(\pi)^{1/2}][\tfrac{10}{3} + (\tfrac{4}{3})b^2] \tag{10}$$

The complementary error function is notated as erfc, and $b$ is defined by

$$b = 2^{-1/2}p(\delta/\Lambda)$$

When $R_L$ is given by the difference of two exponential functions, as shown in Eq. (2), Eqs. (9) or (10) can be used to synthesize the corresponding expression for $R_{\mathrm{opt}}(\delta,\tau)$. Such expressions, obtained with Eqs. (9) and (10), have been compared with the measured $R_{\mathrm{opt}}(\delta,\tau)$ in Fig. 3. The agreement between measured and calculated results is quite good when the ratio of rms to mean velocities is 0.15 to 0.20.

Microwaves scattered by the turbulent flame will have doppler frequency shifts due to both the random and directed velocities in the flame. In the experiment described in Ref. 11, calculations for random velocities with a root mean square value between 10% and 20% of the mean value gave results that agreed well with experimental measurements. It is seen, therefore, that choosing a model having anisotropic electron density correlations and isotropic velocity fluctuations (at approximately 15 to 20% of the value of the mean velocity) provides a good fit to the measured correlations and scattered microwave spectrum of this jet experiment.

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